Dore Primary School, Sheffield

Mathematics Calculation Policy

Overview of calculation strategies - See timetable for suggested introduction (Appendix A)

Early Years into KS1

Practical, oral and mental activities to understand calculation.

Personal methods of recording.

Key Stage 1

Methods of recording / jottings to support calculation (e.g. partitioning)

Introduce signs and symbols ***(+ / - in Year 1 and x / ÷ in Year 2)***

Use images such as empty number lines to support mental and informal calculation.

Year 3

More efficient informal written methods / jottings – expanded methods and efficient use of number lines.

Years 4-6

Continue using efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines. Develop these to larger numbers and decimals where appropriate.

Begin to develop efficient written methods (standard / compact methods) for all four operations

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts

By the end of Year 6, children should:

- have a secure knowledge of number facts and a good understanding of the four operations in order to:
  - carry out calculations mentally when using one-digit and two-digit numbers
  - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**

Children should always **look at the actual numbers (not the size of the numbers**) before attempting any calculation to determine whether or not they need to use a written method.

Therefore, the key question that children should always ask themselves before attempting a calculation is: -

Can I do it in my head?

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a ‘feel’ for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication facts up to 10 × 10 (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2), when applying mental methods in special cases (Year 5);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

Written methods of calculation

The 1999 Framework sets out progression in written methods of calculation that highlights how children would move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations.

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as ‘standard’ written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work. There has been some confusion as to the progression to written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods. The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches which can be beneficial to children. However, if the long-term aim is shared across all schools and if expectations are consistent then children’s progress will be enhanced rather than limited. The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the renewed objectives. Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

Objectives

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

Calculating – Y1-3	**Calculating – Y4-6**
Year 1 • Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number • Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number • Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences	**Year 4** • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6)
Year 2 • Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders • Use the symbols +, –, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. ☐ ÷ 2 = 6, 30 – ☐ = 24)	**Year 5** • Use efficient written methods to add and subtract whole numbers and decimals with up to two places • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 • Refine and use efficient written methods to multiply and divide HTU × U, TU × TU, U.t × U and HTU ÷ U
Year 3 • Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers • Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences	**Year 6** • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

Written methods for addition of whole numbers

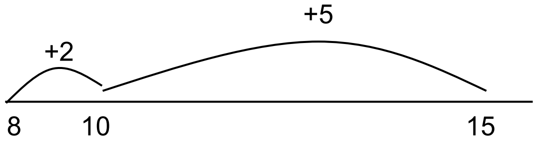
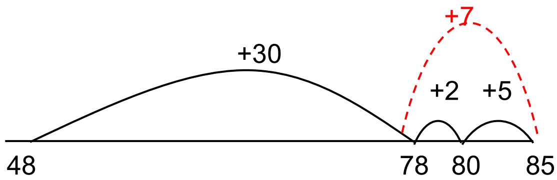
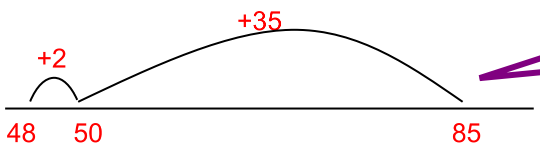
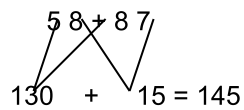
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation for** addition which they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Year group	**Main method**	**Alternative method(s)**
	Stage 1: The empty number line	**Partition one of the numbers**
Year 2 / 3 (Add speech bubbles at start of section – using **‘This is the way we do it”**	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  **48 + 37** = 85  **Alternatives (for some children)** 48 + 37 = 85 	This method will be a jotting approach, and may look like the following examples: - **48 + 37** 48 + 30 = 78 78 + 7 = 85 Or 48 + 30 + 7 = 85 ***Using a number line lets me show my thinking on paper***
Year group	**Main method**	**Alternative method(s)**
	Stage 2: Partitioning	**Partition one of the numbers**
Year 2 / 3 Add speech bubbles Years 4-6	Record steps in addition using partitioning: Initially as a jotting: - **58 + 87** = 50 + 80 + 8 + 7 = 130 + 15 = 145 Or 50 + 80 = 130 8 + 7 = 15 130 + 15 = 145 Partitioned numbers are then written under one another: - 50 8 80 7 130 15 = 145 This method may be appropriate for some children with larger numbers if they struggle with Stages 3-4 500 30 8 / 2400 60 7 200 80 6 / 700 80 5 700 110 14 = 824 / 3100 140 12 = 3252	**58 + 87** ***This method is basically a ‘jotting’ version of the number line method*** Or 87 + 50 = 137 / 58 + 80 = 138 137 + 8 = 145 / 138 + 7 = 145 Or 87 + 50 + 8 = 145 One popular jotting approach is: - 

Stage 3: Expanded method in columns

Year 3

(Simple examples to introduce the expanded method to the children.
Many children would continue to answer these calculations mentally or using a simple jotting –
See **Stage 2**)

A. Single 'carry' in units

Adding the tens first: -

67 + 24

```
   67
 + 24
   80
   11
   91
```

B. 'Carry' in units and tens

58 + 87

```
   58
 + 87
  130
   15
  145
```

'Fifty plus eighty equals one hundred and thirty, because 'five plus eight equals thirteen.

Adding the ones first:

```
   67
 + 24
   11
   80
   91
```

```
   58
 + 87
   15
  130
  145
```

Adding the ones first gives the same answer as adding the tens first

Year 3 / 4

Refine over time to adding the ones digits first consistently, with harder calculations

457 + 76 **538 + 286**

```
   457        Then        538
 +  76                  + 286
    13                     14
   120                    110
   400                    700
   533                    824
```

The time spent practising expanded method will depend on security of number facts recall and understanding of place value.

Stage 4: Column method

Year 4 onwards

Record carry digits below the line

58 + 87 **457 + 76** **538 + 286**

```
   58    Then    457    Then    538
 + 87          +  76          + 286
  123            533            824
  1 1            1 1            1 1
```

Use the words 'carry ten' and 'carry hundred', not 'carry one'

Years 5-6

Once confident, use with larger whole numbers and decimals.
Return to expanded if children make repeated errors

2467 + 785 **4824 + 2369** **46.73 + 78.6**

```
   2467        4824        46.73
 +  785      + 2369        78.60
   3252        7193       125.33
   1 1 1       1  1       1 1 1
```

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for** subtraction which they know they can rely on **when mental methods are not appropriate.**

But, they should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 2006 -128 then the 'counting on' approach may well be the best method in that particular instance

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies: -

Taking away (Counting Back) ***Complementary Addition (Counting On)***

When should we count back and when should we count on?

This will alter depending on the calculation (see below), but often the following rules apply

If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

If the numbers are close together (206 – 188), then count up

In many cases, either strategy would be suitable

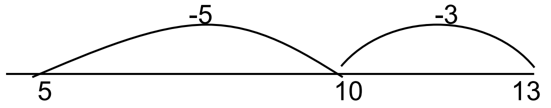
Year group	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
	Stage 1: Using the empty number line	

The empty number line helps to record or explain the steps in mental subtraction.
It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly.
It can also show **counting up** from the smaller to the larger number to **find the difference,**

Year 2

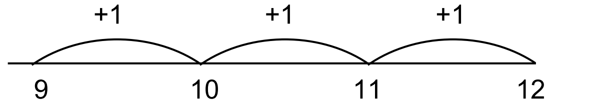
The steps often bridge through a multiple of 10.

13 – 8 = 5



Small differences can be found by counting up

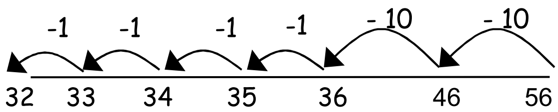
12 – 9 = 3



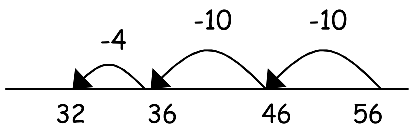
Year 2/3

For 2 digit numbers, count back in 10s and 1s

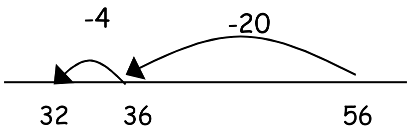
56 – 24 = 32



Then subtract the units in a single jump



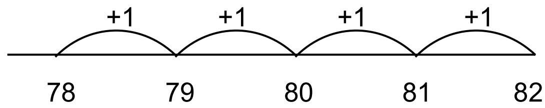
Then subtract tens and units in single jumps



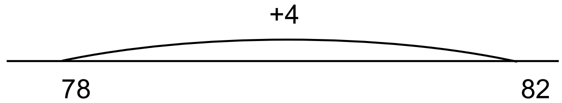
For 2 (or 3) digit numbers close together, count up

82 – 78 = 4

First, count in ones



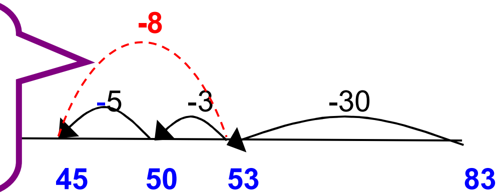
Then, use number facts to count in a single jump



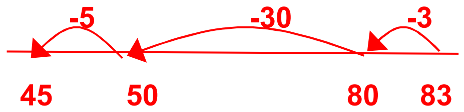
Some numbers (83 – 38) can be subtracted just as quickly either

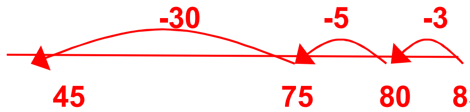
83 – 38 = 45

Partition 38. Take away 30 then take away 8 (-3 -5)

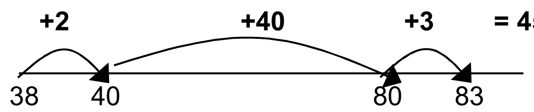


Alternatives





Count up from the smaller to the larger number.



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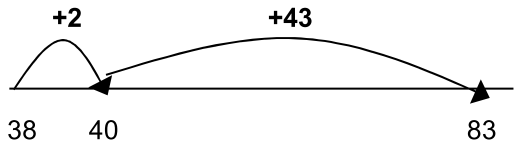


	Stage 2: Subtraction by counting back Expanded method	Subtraction by counting up Number lines (continued)

Year 3 / 4

Introduce the expanded method with 2 digit numbers to explain the process.
Partition both numbers into tens and ones.
Exchange from the tens to the ones.

83 – 38

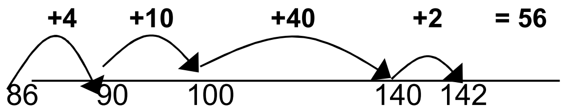
```
   80   3        70  13        70    1
-  30   8        30   8        8̶0̶    3
                            -  30    8
                               40    5
```

Exchange from hundreds to tens and tens to ones

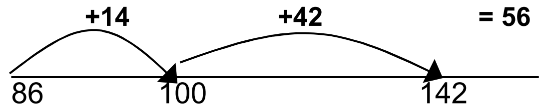
142 - 86

```
100  40  2      100 30 12      130  1
-    80  6          80  6      1̶0̶0̶  4̶0̶  2
                                    80  6
                                    50  6
```

142 – 86



Or (in fewer steps)



Year 4

Take the method into three digit numbers
Subtract the ones then the tens then the hundreds
Demonstrate without exchanging first

A

784 - 351

```
   700   80   4
-  300   50   1
   400   30   3
```

For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.

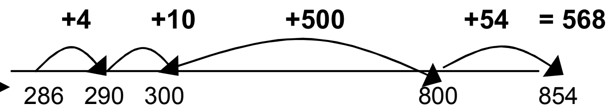
B

Move towards exchanging from hundreds to tens and tens to ones

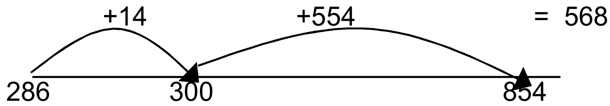
854 - 286

```
   800   50   4        700   140   1
-  200   80   6        8̶0̶0̶   5̶0̶    4
                     - 200   80    6
                       500   60    8
```

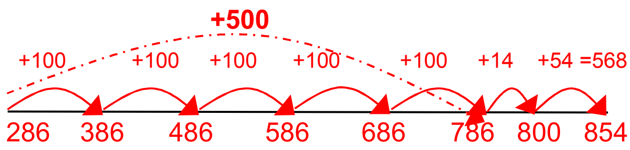
854 - 286



Or (the efficient method)



Alternative (count the hundreds first)



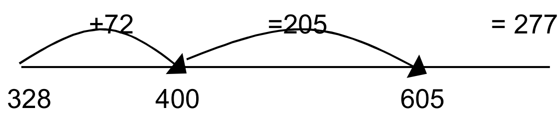
C

Use some examples which include the use of zeros

605 – 328

```
   600   0   5         500  90   1
-  300  20   8              1̶0̶0̶
                       6̶0̶0̶   0̶    5
                     - 300  20    8
                       200  70    7
```

For numbers containing zeros, counting up is often the most reliable method.



Continue to use expanded subtraction until both number facts and place value are considered to be very secure

Stage 3: Standard method (decomposition)

Mainly Y5 onwards

Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.

(Using example B from Stage 2)

854 – 286

```
 7  14  1
 8̸  5̸   4
-2   8   6
 5   6   8
```

Continue to refer to digits by their **actual** value, not their digit value, when explaining a calculation. E.g. One hundred and forty subtract eighty.

Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)

(Using example C from Stage 2)

605 – 328

```
    9
 5  1̸  1
 6̸  0   5
-3  2   8
 2  7   7
```

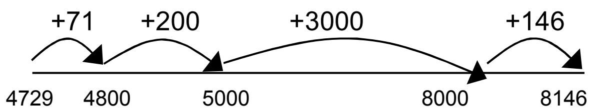
The counting up method is ofte nused in Years 5 and 6 for children whose mental recall is weak, or who require a visual image to support their thinking.

Move onto examples using 4 digit (or larger) numbers and then onto decimal calculations.

814 6 – 472 9

```
 7  1  3  1
 8̸  1  4̸  6
-4  7  2  9
 3 4   1  7
```

814 6 – 472 9



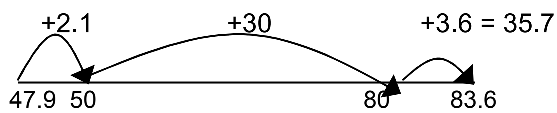
= 3000
146
200
71
341 7

Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.

83.6 – 47.9

```
  7  12  1
  8̸  3̸.  6
- 4̸  7.  9
  3  5.  7
```

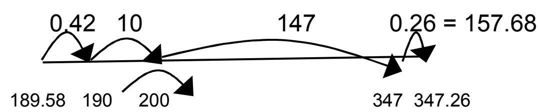
83.6 – 47.9



347.26 – 189.58

```
  1  13  16   11  1
  3̸  4̸   7̸ .  2̸  6
-  1̸  8   9 .  5  8
   1  5   7 .  6  8
```

347.26 – 189.58



Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 10 × 10;**
- **partition number into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note:

Children need to acquire **one efficient written method of calculation for** multiplication which they know they can rely on **when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

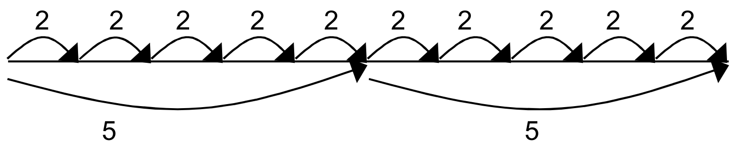
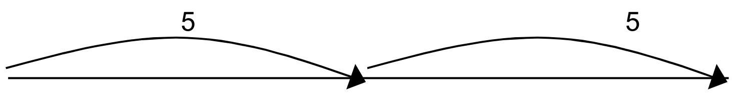
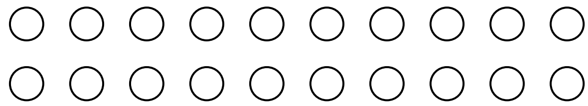
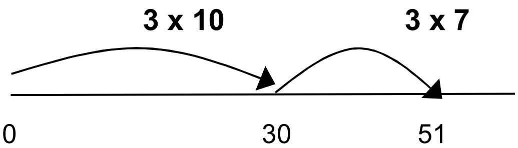
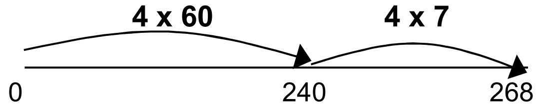
These mental methods are often more efficient than written methods when multiplying.

Use partitioning and grid methods until number facts and place value are secure

For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

When multiplying a 2 digit x 3 digit number (or a 3digit x 3 digit number), the standard method is usually the most efficient

At all stages, use known facts to find other facts. E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16)

	Expanded multiplication	Standard 'compact' multiplication
Year group	**Stage 1: Number lines and mental methods**	
Year 2	Begin by building on the understanding that multiplication is repeated addition, using arrays and number line s to support the thinking. **Using a number line** 2 x 10 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2  Or 10 x 2 = 10 + 10 **2 x 10 = 10 x 2**  **Using an array**  **10 x 2 = 20** **2 x 10 = 20**	
Year 3 **Years 3-4**	Extend the above methods to include the 3, 4 and 6 times tables then begin to partition using **jottings and number lines.** **3 x 17** **x 3** **10** **+** **7** ↓ ↓ 30 + 21 = 51  becomes  Or 10 x 3 = 30 7 x 3 = <u>21</u> 51 Extend the methods above to calculations which give products greater than 100. **4 x 67** **x4** 60 + 7 **x 4** ↓ ↓ 240 + 28 = 268  Or 60 x 4 = 240 7 x 4 = <u>28</u> 268 ***Each of these methods can be used in the future if children find expanded or standard methods difficult*** Extend to using these methods with all tables to 10 x 10.	

Year group	Stage 2: Written methods – Short multiplication	
	Grid multiplication	**Vertical multiplication (Expanded method into standard)**

Late Year 3 onwards (Mainly Year 4)

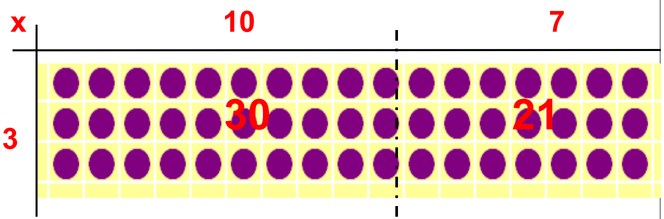
Grid multiplication

The grid method of multiplication is a simple, alternative way of recording the jottings shown previously.

3 x 17

	10	7	
3	30	21	= 51

If necessary (for some children) it can initially be taught using an array to show the actual product.



Vertical multiplication

The expanded method links the grid method to the standard method.

It still relies on partitioning the tens and units, but sets out the products vertically.

Children will use the expanded method until they can securely use and explain the standard method.

When setting out calculations vertically, begin with the units first (as with addition and subtraction)

Year 4 / 5

Grid multiplication

4 x 67

	60	7	
4	240	28	= 268

Use all tables with more complex calculations

7 x 89

	80	9	
7	560	63	= 623

Move onto HTU x U

4 x 378

	300	70	8
4	1200	280	32

= 1512

The grid method may continue to be the main method used by children whose mental and written calculation skills are weak, or children who need the visual link to place value.

Vertical multiplication

4 x 67

```
  6 7              6 7
x   4    →       x   4
  2 8              2 6 8
2 4 0              2
2 6 8
```

Place the 'carry' digit below the line

7 x 89

```
  8 9              8 9
x   7    →       x   7
  6 3            6 2 3
5 6 0              6
6 2 3
```

Where numbers are difficult to add mentally, try to use the expanded or standard methods

4 x 378

```
  3 7 8            3 7 8
x     4   →      x     4
    3 2          1 5 1 2
  2 8 0            3 3
1 2 0 0
```

In all calculations, refer to the actual value of the digits.
E.g. 4 multiplied by by 70, not 7

Stage 3: Long multiplication: TU x TU

Year group	Grid long multiplication	Vertical 'standard' long multiplication
Years 5 & 6	Extend the grid method to TU × TU, asking children to estimate first. '	**Children should only use the 'standard' method of long multiplication if they can regularly get the correct answer using this method.**

Years 5 & 6 — Grid long multiplication

38 x 57

38 × 57 is approximately 40 × 60 = 2400.

x	50	7	
30	1500	210	**1 7 1 0**
8	400	56	**4 5 6**
			2 1 6 6
			1

Add the two products in each row

Add these sums for the overall product

The grid method is often the 'choice' of many children in Years 5 and 6, and is the method that they will mainly use for long multiplication.

Years 5 & 6 — Vertical 'standard' long multiplication

38 x 57

38 × 57 is approximately 40 × 60 = 2400.

```
   3 8                3 8
 x 5 7     or   x    5 7
                     2 5
 2 6 6              2 6 6
  2 5               1 4
1 9 0 0           1 9 0 0
 1 4
2 1 6 6           2 1 6 6
```

There is no 'rule' regarding the position of the 'carry'digits. Each choice has advantages and complications.

Either carry the digits mentally or have your own favoured position for these digits.

Stage 4: Long multiplication: HTU x TU

Year 6 — Grid long multiplication

For HTU x TU, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

Use this method when you find the standard method to be unreliable or difficult.

423 x 68

423 × 68 is approximately 400 × 70 = 28000.

X	60	8	
400	24000	3200	**27200**
20	1200	160	**1360**
3	180	24	**204**
			28764

Year 6 — Vertical 'standard' long multiplication

Many children working at Level 5 choose the standard method. For HTU x TU calculations It especially efficient, and less prone to errors when mastered.

423 x 68

423 × 68 is approximately 400 × 70 = 28000.

```
    4 2 3               4 2 3
    x 6 8     or   x      6 8
                          1 2
  3 3 8 4             3 3 8 4
    1 2                1 4
2 5 3 8 0           2 5 3 8 0
   1 1
2 8 7 6 4           2 8 7 6 4
    1                   1
```

As before, either carry the 'carry'digits mentally or decide on your own favoured position for them.

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for** subtraction which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

- **understand division as repeated subtraction;**
- **estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;**
- **multiply a two-digit number by a single-digit number mentally;**
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)**
- **subtract numbers using the column method.**

The above points are crucial. If children do not have a secure understanding of these prior learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.

72 ÷ 6 'chunks' into 60 and 12

72 ÷ 4 'chunks' into 40 and 32

72 ÷ 3 'chunks' into 30 and 42 (or 30, 30 and 12)

Stage 1: Number line division and mental division (pre chunking)

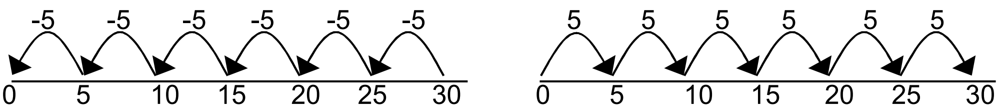
Year group

Year 3

Start to emphasise grouping over sharing as a more efficient way to divide.

Beginning with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, and as counting forward to find how many times one number 'goes into' another.

30 ÷ 5 or

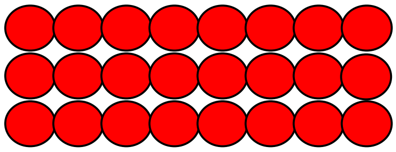


It also helps the children to deal with remainders.

23 ÷ 5 = 4 r 3 or

Regularly stres the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.

Some children will continue to use arrays to develop their understanding of division, and to link to multiplication facts.

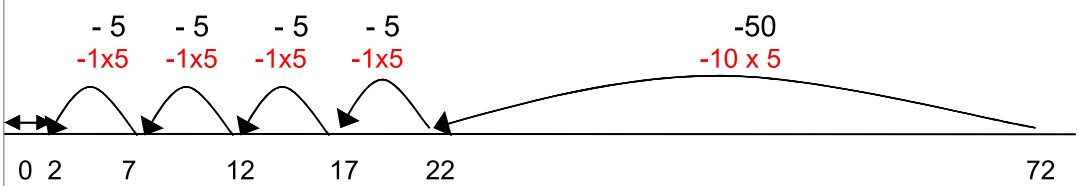


This array can show 24 ÷ 3 and 24 ÷ 8

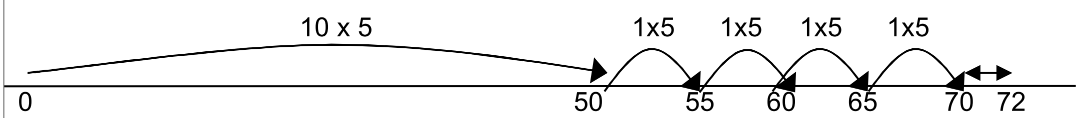
Years 3 and 4

The number line is also an excellent way of introducing the 'chunking' approach.

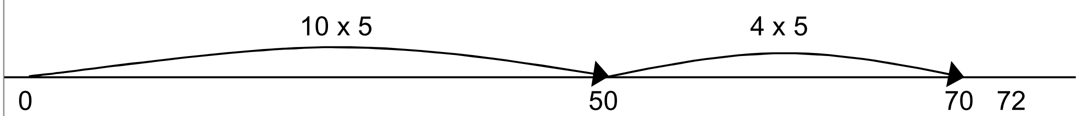
72 ÷ 5 = 14 r 2



Or



Into a more efficient



In lower KS2, children need a great deal of practice in mentally 'chunking' to develop their understanding of division. They can use an informal jotting to support their thinking.

These mental methods for dividing TU ÷ U are usually based on partitioning in different ways.

72 ÷ 5

72 ÷ 5 = 14 r 2

50 20 2

10x5 4x5

72 ÷ 6

72 ÷ 6 = 12

60 12

10x6 2x6

72 ÷ 4

72 ÷ 4 = 13

40 32

10x4 8x4

72 ÷ 3

72 ÷ 3 = 24

60 12

20x3 4x3

Stage 2: Short division 'chunking'

Year group	
Year 4	'Short' division of **TU ÷ U** introduces the 'chunking' method. This becomes more useful with **HTU ÷ U** and later for long division.

Once children can understand chunking for TU ÷ U, they can move on to HTU ÷ U quite quickly.

When chunking we repeatedly subtract multiples or 'chunks' of the divisor

- This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding children of the link between division and repeated subtraction.

However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

```
6)196
 − 60   6×10
  136
 − 60   6×10
   76
 − 60   6×10
   16
 − 12   6× 2
    4     32
Answer:   32 R 4
```

Stage 3: ‘Expanded’ method for HTU ÷ U

<table>
<tr><th colspan="2">Stage 3: ‘Expanded’ method for HTU ÷ U</th></tr>
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The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that ‘trap’ the HTU dividend.
Estimating has two purposes when doing a division:

to help to choose a starting point for the division;
to check the answer after the calculation.

Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

</td><td>
To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.
Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.
$$\begin{array}{rrl} & 6\overline{)196} & \\ - & \underline{180} & 6\times 30 \\ & 16 & \\ - & \underline{12} & 6\times \underline{2} \\ & 4 & 32 \\ \text{Answer:} & & 32\,\text{R}\,4 \end{array}$$
The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.
</td></tr>
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Stage 4: Short division of HTU ÷ U

Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

- For most children this will be at the end of Year 4 or the beginning of Year 5.
- The accompanying patter is ‘How many threes divide into 80 so that the answer is a multiple of 10?’ This gives 20 threes or 60, with 20 remaining. We now ask: ‘What is 21divided by three?’ which gives the answer 7.

For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21. Each number is then divided by 3.

$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

The short division method is recorded like this:

$$\begin{array}{r} 20 + \;\, 7 \\ 3\overline{)60 + 21} \end{array}$$

This is then shortened to:

$$\begin{array}{r} 2\;7 \\ 3\overline{)8\;{}^{2}1} \end{array}$$

The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.

- The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.

‘Short’ division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.

- The accompanying patter is ‘How many threes in 290?’ (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: ’How many threes in 21?’ which has the answer 7.
- Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.
- For most children this will be at the end of Year 5 or the beginning of Year 6.

For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3.

$$\begin{aligned} 291 \div 3 &= (270 + 21) \div 3 \\ &= (270 \div 3) + (21 \div 3) \\ &= 90 + 7 \\ &= 97 \end{aligned}$$

The short division method is recorded like this:

$$3\overline{)290 + 1} = \begin{array}{r} 90 + \;\, 7 \\ 3\overline{)270 + 21} \end{array}$$

This is then shortened to:

$$\begin{array}{r} 9\;7 \\ 3\overline{)2\;9\;{}^{2}1} \end{array}$$

The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3.

The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.

Stage 5: Long division

- The next step is to tackle HTU ÷ TU, which for most children will be in Year 6.
- The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.
- Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

```
24 ) 560
20  – 480      24 × 20
       80
 3     72      24 × 3
        8
```

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

```
       23
24 )  560
     –480
       80
      –72
        8
```

Answer: 23 R 8